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NEW SOCIABLE NUMBERS

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ABSTRACT. An exhaustive search has yielded new sociable groups; one of order 9, two of order 8, and the others of order 4.

For each natural number n, we write $s(n) = \sigma(n) - n$ for the number of its divisors excluding itself. If this function is iterated by $s^{j+1}(n) = s(s^j(n))$, it defines the so-called aliquot sequence of n: $s^0(n), s^1(n), s^2(n), \dots$, starting with $s^0(n) \equiv n$. If the sequence for a given n is bounded, either it ends at 0 (since $s(0)$ is undefined), or it becomes periodic. If it is constant, it has reached a perfect number. If it is alternating, it represents a pair of amicable numbers, or in general produces after k iterations a cycle $s^{k+1}(n), s^{k+2}(n), \dots, s^{k+t}(n)$ of minimal length t, which forms a sociable group of order t.

There is a concise historical survey on the search for perfect numbers in [1], and thousands of amicable pairs are known today [2], but much less is known about groups of higher order. At the beginning of this century, the first two examples, order 5 and order 28, were found by Poulet [3]. In 1969 and 1970, Borho [4] and Cohen [5] discovered some of order 4. This work was extended some years later by Devitt et al. [6] and Root [7], who found five further groups of order 4.

In order to find more examples, I initiated a search for sociable numbers on several computers. Testing the first t iterates of each number n, I used about 10000 cpu hours on HP320/HP330 computers.

limit of n	max order of t
$5 \cdot 10^4$	50
$5 \cdot 10^5$	40
$5 \cdot 10^6$	30
$5 \cdot 10^7$	20
$5 \cdot 10^8$	10
$5 \cdot 10^9$	30*

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* In order to save cpu time, I broke off the search in the range over $5 \cdot 10^8$ if the iterate exceeded six times the starting value of the sequence.

The main result is the discovery of 11 previously unknown sociable groups which are shown with their factorization in the following table; in addition, I reproduced the 1100 amicable pairs computed by Riele [2].

$1236402232 = 2 \cdot 2 \cdot 2 \cdot 13 \cdot 41 \cdot 53 \cdot 5471$
$1369801928 = 2 \cdot 2 \cdot 2 \cdot 11 \cdot 17 \cdot 863 \cdot 1061$
$1603118392 = 2 \cdot 2 \cdot 2 \cdot 313 \cdot 640223$
$1412336648 = 2 \cdot 2 \cdot 2 \cdot 4967 \cdot 35543$

$1799281330 = 2 \cdot 5 \cdot 7 \cdot 11 \cdot 139 \cdot 16811$
$2267877710 = 2 \cdot 5 \cdot 7 \cdot 32398253$
$2397470866 = 2 \cdot 7 \cdot 17 \cdot 10073407$
$1954241390 = 2 \cdot 5 \cdot 19 \cdot 73 \cdot 140897$

$2387776550 = 2 \cdot 5 \cdot 5 \cdot 19 \cdot 31 \cdot 89 \cdot 911$
$2497625050 = 2 \cdot 5 \cdot 5 \cdot 19 \cdot 31 \cdot 84809$
$2550266150 = 2 \cdot 5 \cdot 5 \cdot 31 \cdot 59 \cdot 79 \cdot 353$
$2506553050 = 2 \cdot 5 \cdot 5 \cdot 31 \cdot 59 \cdot 27409$

$2717495235 = 3 \cdot 3 \cdot 5 \cdot 7 \cdot 13 \cdot 19 \cdot 53 \cdot 659$
$3509525565 = 3 \cdot 3 \cdot 5 \cdot 7 \cdot 13 \cdot 857027$
$3977471043 = 3 \cdot 3 \cdot 7 \cdot 13 \cdot 1451 \cdot 3347$
$3100575933 = 3 \cdot 3 \cdot 7 \cdot 13 \cdot 19 \cdot 19 \cdot 10487$

$2879697304 = 2 \cdot 2 \cdot 2 \cdot 11 \cdot 19 \cdot 1722307$
$3320611496 = 2 \cdot 2 \cdot 2 \cdot 17 \cdot 71 \cdot 343891$
$3364648984 = 2 \cdot 2 \cdot 2 \cdot 31 \cdot 13567133$
$3147575336 = 2 \cdot 2 \cdot 2 \cdot 47 \cdot 8371211$

$3705771825 = 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 1019 \cdot 2309$
$3890616975 = 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 503 \cdot 1637$
$4298858865 = 3 \cdot 3 \cdot 3 \cdot 5 \cdot 7 \cdot 79 \cdot 89 \cdot 647$
$4659093135 = 3 \cdot 3 \cdot 3 \cdot 5 \cdot 101 \cdot 341701$

$4424606020 = 2 \cdot 2 \cdot 5 \cdot 41 \cdot 103 \cdot 52387$
$5186286908 = 2 \cdot 2 \cdot 11 \cdot 1861 \cdot 63337$
$4720282996 = 2 \cdot 2 \cdot 11 \cdot 13 \cdot 1301 \cdot 6343$
$4993345292 = 2 \cdot 2 \cdot 13 \cdot 1291 \cdot 74381$

$4823923384 = 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 1087 \cdot 11321$
$5708253896 = 2 \cdot 2 \cdot 2 \cdot 23 \cdot 211 \cdot 147029$
$5513075704 = 2 \cdot 2 \cdot 2 \cdot 67 \cdot 97 \cdot 107 \cdot 991$
$5196238856 = 2 \cdot 2 \cdot 2 \cdot 37 \cdot 743 \cdot 23627$

$1095447416 = 2 \cdot 2 \cdot 2 \cdot 7 \cdot 313 \cdot 62497$
$1259477224 = 2 \cdot 2 \cdot 2 \cdot 43 \cdot 3661271$
$1156962296 = 2 \cdot 2 \cdot 2 \cdot 7 \cdot 311 \cdot 66431$
$1330251784 = 2 \cdot 2 \cdot 2 \cdot 43 \cdot 3867011$
$1221976136 = 2 \cdot 2 \cdot 2 \cdot 41 \cdot 1399 \cdot 2663$
$1127671864 = 2 \cdot 2 \cdot 2 \cdot 11 \cdot 61 \cdot 83 \cdot 2531$
$1245926216 = 2 \cdot 2 \cdot 2 \cdot 19 \cdot 8196883$
$1213138984 = 2 \cdot 2 \cdot 2 \cdot 67 \cdot 2263319$

$1276254780 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 1973 \cdot 10781$
$2299401444 = 2 \cdot 2 \cdot 3 \cdot 991 \cdot 193357$
$3071310364 = 2 \cdot 2 \cdot 767827591$
$2303482780 = 2 \cdot 2 \cdot 5 \cdot 67 \cdot 211 \cdot 8147$
$2629903076 = 2 \cdot 2 \cdot 23 \cdot 131 \cdot 218213$
$2209210588 = 2 \cdot 2 \cdot 13 \cdot 13 \cdot 17 \cdot 192239$
$2223459332 = 2 \cdot 2 \cdot 131 \cdot 4243243$
$1697298124 = 2 \cdot 2 \cdot 907 \cdot 467833$

$805984760 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 7 \cdot 1579 \cdot 1823$
$1268997640 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 17 \cdot 61 \cdot 30593$
$1803863720 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 103 \cdot 367 \cdot 1193$
$2308845400 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 11544227$
$3059220620 = 2 \cdot 2 \cdot 5 \cdot 2347 \cdot 65173$
$3367978564 = 2 \cdot 2 \cdot 841994641$
$2525983930 = 2 \cdot 5 \cdot 17 \cdot 367 \cdot 40487$
$2301481286 = 2 \cdot 13 \cdot 19 \cdot 4658869$
$1611969514 = 2 \cdot 805984757$

In particular, a question of Meissner [4] is answered positively, concerning the existence of sociable groups of order 8 and 9.

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